Optimal Power Allocation in Spatial MIMO Channel using Heuristic Algorithms

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Received: June 2015 Revised: July 2015 Accepted: August 2015

ABSTRACT:
The topic of power allocation in multiple-input-multiple-output (MIMO) systems for wireless communication in order to reach high capacity or low bit error rate has gained some attention. In this paper, heuristic algorithms, including genetic and particle swarm optimization algorithms, are applied to find the optimal power allocation for achieving best capacity benefit. Two cases of un-polarized and cross-polarized antennas of spatial MIMO channel modeling are studied. We demonstrate that the performance of genetic and particle swarm optimization algorithms is optimal compared with a perfect search at a reduced computational complexity. These algorithms have fast convergence and can handle large number of sub-channels without performance degradation. Both simulation and numerical results confirm that, compared to other mathematical methods, the proposed algorithms are more efficient in terms of complexity and power assignment to antennas in a MIMO system.

KEYWORDS: MIMO systems, channel capacity, genetic algorithm, particle swarm optimization algorithm, cross-polarized antennas, spatial channel model.

1. INTRODUCTION
A multiple-input-multiple-output (MIMO) system can provide different paths between transmitter and receiver through various sub-channels [1]. If one path is simply used to send a signal, it is likely that this channel (signal) undergoes a deep fade. By transmitting repetitive data via different paths, however, multiple independently faded copies of the data symbols are taken at the receiver and, as a result, more reliable detection can be achieved. In addition, signal reception from different sub-channels increases channel capacity [1].

Systems based on MIMO are best candidates to be used in modern communication systems. Radio propagation model of these systems has provided chief characteristics including diversity gain, enhanced network flexibility, boosted power gain as a result of transmit beam-forming, increased degree of freedom, and raised capacity [1], [2].

Three accepted modeling techniques used for MIMO channels include ray tracing model, correlation model, and scattering model. In the third model, also known as spatial channel model (SCM), a distinct distribution of scatterers is assumed and the channel is modeled accordingly. Therefore, the statistical features of the channel can be selected in a way to represent various environments of MIMO channels. Since the wireless propagation medium is not explicitly modeled, its complexity is less than that of two other models. On the other hand, some disadvantages of this model are complexity in parameters, scattering distribution construction for various channel environments, and inclusion of high number of parameters [3].

Advantages of cross-polarized antennas in advanced cellular systems in reducing array length and increasing capacity [4] have led us to use this feature in MIMO channel modeling. In two-dimensional (2-D) spatial channel modeling, disregarding elevation spectrum, numerous effects of electromagnetic propagation in outdoor environments can still be included [4].

In propagation environments which focus on angular spectrum, the assumption of 2-D propagating waves is not efficient. In these cases, erroneous results are obtained by just considering azimuth spectrum. So in order to achieve desired results, the 2-D model is corrected and elevated to the next three dimensional (3-D) model. In this article, we just present the 2-D channel modeling.

Genetic algorithm (GA) is a calculation model inspired by evolution which encodes a potential solution to a definite problem on an uncomplicated chromosome similar to data structure, and exerts recombination
operators to these structures in such a way as to maintain vital information. The GA is frequently observed as function optimizer, though the range of problems to which the GA has been exerted is reasonably extensive. An implementation of a GA commences with a population of (usually random) chromosomes. One the examined these structures and assigns reproductive occasion in such a way that those chromosomes which indicate a better result to the objective problem are given more chances to regenerate than those chromosomes which are not as good as solutions. The good quality of a solution is usually determined according to the present population.

Particle swarm optimization (PSO) is a population-based stochastic optimization technique inspired by social activities of animals, such as bird flocking or fish schooling. The PSO has many similarities with the GA. The scheme is initialized with a population of random solutions and looks for optimum values by updating generations. On the other hand, unlike the GA, the PSO has no evolution operators such as crossover and mutation. The fundamental concept of the PSO algorithm is to generate a swarm of particles which move around in the search space according to uncomplicated mathematical formula over the particles position and velocity. The PSO is easily fulfilled and needs a few parameters to regulate. These are some of the benefits of the PSO compared to the GA.

Recent work on the application of the GA and PSO in MIMO system is explained as follows. In [5], the design of a MIMO antenna system is presented and the GA is proposed to obtain position and orientation of each array antenna that maximizes capacity. Also, a uniform linear array, a uniform circular array, and a GA-optimized array are compared therein. In [6], a technique for array optimization of MIMO radar has been investigated where the GA has been applied to array optimization to reduce side-lobe peaks by acting on the position of antenna elements. Also in [7], authors use the PSO in order to reduce side-lobe peaks of MIMO radar. In [8], the PSO is employed to choose optimal active antenna configuration satisfying the capacity requirement while at the same time led to minimize the energy consumption in MIMO systems. In [9], exerting GA to arrangement of antennas has led to optimum performance of users. In [10], the problem was how to design the position of multiple antennas on wireless terminal. Therefore, GA was employed to find optimal antenna placement in order to achieve the reasonable capacity. In [11], the PSO is used as an estimator to get good channel estimation. In [12], authors propose a hybrid GA for joint quantized precoding and transmit antenna selection to get maximum capacity. Their target is to mitigate the effect of multi-user interference and to reduce hardware costs. In [13], the PSO is used to synthesis radiation pattern of a directional circular arc array to maximize the capacity performance in an indoor MIMO ultra-wide band. In [14], the PSO based beams optimization algorithm is proposed to maximize cell spectral efficiency. Hence, power allocation of two vertical beams is addressed.

Most of these previous works concentrated on array arrangement of MIMO antennas using GA or PSO to attain their goals. But, we have assigned optimal power on uniform linear antennas array to get maximum capacity.

This paper is organized as follows. The required parameters in spatial MIMO channel modeling, without considering cross-polarized antennas, are studied and then the steps of channel modeling are introduced. Subsequently, by considering cross-polarized antennas and introducing new parameters, the original channel model is improved. In the next step, we examine the capacity of the system. Then, we have used genetic and particle swarm optimization algorithms to assign antennas power in order to obtain the maximum capacity of the channel. After that, the applied parameters in simulation are introduced and implemented for each models. In addition, the capacity for each models are calculated using waterfilling, genetic, and particle swarm optimization algorithms.

2. PRIMARY MODEL WITHOUT CROSS-POLARIZED ARRAYS

The aim of this section is recognition of necessary parameters for spatial and temporal channel, which consider a single base station transmitting to a single mobile station. Furthermore, cross-polarized arrays have not been applied in the modeling.

2.1. Description of channel model

The overall procedure for generating the channel matrix consists of three basic steps. The first one specifies an environment such as suburban macro, urban macro or urban micro, since every environment has its own features that are different from other environments. In the second step, parameters associated with the environment are obtained to be used in simulations. In the last step, channel coefficients are generated on the basis of required parameters.

The received signal at the MS is composed of exact copies of \( N \) time-delayed multipath, associated with the transmitted signal. These \( N \) paths have been defined by different powers and delays, and chosen randomly according to the channel generation procedure. Additionally, each path consists of \( M \) sub-paths [3], [15], [16], [17].

Angular parameters, used in channel modeling are shown in Fig. 1.
These parameters are defined as in the following:

- \( \Omega_{BS} \): direction of BS antenna array.
- \( \theta_{BS} \): angle between the BS-MS line of sight (LOS) and the BS broadside.
- \( \delta_{n,AoD} \): angle of departure (AoD) for the \( n^{th} \) path.
- \( A_{h,m,AoD} \): offset for the \( m^{th} \) sub-path of the \( n^{th} \) path.
- \( \theta_{h,m,AoD} \): AoD for the \( m^{th} \) sub-path of the \( n^{th} \) path.
- \( \Omega_{MS} \): MS antenna array direction.
- \( \theta_{MS} \): angle between the BS-MS line of sight (LOS) and the MS broadside.
- \( \delta_{n,AoA} \): angle of arrival (AoA) for the \( n^{th} \) path.
- \( A_{h,m,AoA} \): offset for the \( m^{th} \) sub-path of the \( n^{th} \) path.
- \( \theta_{h,m,AoA} \): AoA for the \( m^{th} \) sub-path of the \( n^{th} \) path.
- \( \theta_v \): angle of the velocity vector.

### 2.2. Generation of channel matrix

Channel coefficients are generated using parameters in Section 2.1. The numbers of MS and BS antennas are \( S \) and \( U \), respectively. So, the matrix of the channel coefficients for each of the \( N \) paths is given by means of a \( U \times S \) matrix, which is shown by \( H_{,s}(t) \). The \((u,s)\) component of this matrix is expressed as [3], [17]:

\[
h_{u,s,n}(t) = \sqrt{P_\text{u} \sigma_{SF} / M} \times \exp\left( 2\pi d_u \sin\left( \theta_{h,m,AoD} / \lambda \right) + \phi_{h,m} \right) \times \exp \left( 2\pi d_s \sin\left( \theta_{h,m,AoA} / \lambda \right) \right) \times \exp \left( 2\pi |v| \cos\left( \theta_{h,m,AoA} - \theta_v / \lambda \right) \right)
\]

where:
- \( \sigma_{SF} \): log-normal shadow fading (SF).
- \( \lambda \): wavelength in meters.
- \( d_s \): distance from \( s^{th} \) element of BS antenna to the reference antenna in meters.
- \( d_u \): distance from \( u^{th} \) element of MS antenna to the reference antenna in meters.
- \( \phi_{h,m} \): phase of the \( m^{th} \) sub-path of the \( n^{th} \) path.
- \( |v| \): magnitude of the MS velocity vector.
- \( P_\text{u} \): power of the \( n^{th} \) path.
- \( M \): number of sub-paths per path.
- \( G_{BS}(\theta_{h,m,AoD}) \): BS antenna gain.
- \( G_{MS}(\theta_{h,m,AoA}) \): MS antenna gain.

The parameters used in (1) are defined as following:

- \( \Omega_{BS} \): direction of BS antenna array.
- \( \Omega_{MS} \): direction of MS antenna array.
- \( \lambda \): wavelength in meters.
- \( d_s \): distance from \( s^{th} \) element of BS antenna to the reference antenna in meters.
- \( d_u \): distance from \( u^{th} \) element of MS antenna to the reference antenna in meters.
- \( \phi_{h,m} \): phase of the \( m^{th} \) sub-path of the \( n^{th} \) path.
- \( |v| \): magnitude of the MS velocity vector.

The \((u,s)\) component of the channel matrix indicates path gain from \( s^{th} \) element of BS antenna to \( u^{th} \) element of MS antenna and is obtained from:

\[
h_{u,s,n}(t) = \sum_{m=1}^{M} h_{u,s,m,n}(t)
\]

So the channel matrix from transmitter to receiver can be expressed as:

\[
H(t) = \begin{bmatrix} h_{1,1}(t) & \cdots & h_{1,U}(t) \\ \vdots & \ddots & \vdots \\ h_{U,1}(t) & \cdots & h_{U,S}(t) \end{bmatrix}_{U \times S}
\]

### 3. SCM USING CROSS-POLARIZED ANTENNAS

Cross-polarized antennas in advanced cellular systems become popular gradually. Hence, this feature has been used in MIMO systems in order to reach high capacity and decrease length of the BS and MS arrays.

We consider introducing parameters in MIMO channel modeling using cross-polarized antennas. In the SCM model, mentioned in this section, vertical spectrum has not been considered, and 2-D spatial channel modeling has been introduced.

In this model, the polarization is decomposed into vertical and horizontal directions. The components with analogous polarity are mixed with each other, but
others have less mixing. So, four channels have been considered between MS and BS antennas. These channels show the relationship between components with horizontal and vertical polarity in BS array, and components with horizontal and vertical polarity in MS array. Therefore, antenna patterns in MS and BS have been considered into vertical and horizontal directions [4]. The model presented in this section has original step similar to Section 2, however, the polarity components have been added.

3.1. Description of 2-D channel model

The \( P_2 \) power of each path in horizontal direction depends on the \( P_1 \) power in vertical direction that is defined as cross-polarization discrimination by \( XPD=P_1/P_2 \). The whole \( M \) sub-paths of the \( n \)th path have same \( XPD \), but each path has independent \( XPD \) [3], [4].

For this section, ideal tilted dipole antennas have been assumed. Mixing of horizontal and vertical components arises from path effects, and antenna polarization outflow effects are neglected.

Ideal dipole antenna with polarization vector \( p \) and \( \alpha \) angle from \( z \) axis has vertical and horizontal components corresponding to [4]:

\[
\chi(k) = \begin{bmatrix} \chi^v(k) \\ \chi^h(k) \end{bmatrix} = \begin{bmatrix} \cos \alpha \\ \sin \alpha \cos \phi \end{bmatrix} \exp(jkr)
\]

(4)

Where, vector \( r \) is the distance of antenna from the center of the antenna array and vector \( k \) is 2-D wave vector with the carrier wavelength \( \lambda \) and azimuth angle \( \phi \). This vector, \( k \), is defined as:

\[
k = \frac{2\pi}{\lambda} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}
\]

(5)

According to (4), complex response of the BS antenna for horizontal and vertical components is obtained from:

\[
\chi_{BS}(k) = \begin{bmatrix} \cos \alpha \\ \sin \alpha \cos(\theta_{n,m,\lambda \phi}) \end{bmatrix} \exp(jkr)
\]

(6)

Where, the first element is the complex response of the BS antenna for vertical component, and the second element is the complex response of the BS antenna for horizontal component.

Using (4), the complex response of the MS antenna for a wave component with vertical and horizontal polarity is obtained from:

\[
\chi_{MS}(k) = \begin{bmatrix} \cos \alpha \\ \sin \alpha \cos(\theta_{n,m,\lambda \phi}) \end{bmatrix} \exp(jkr)
\]

(7)

In which, the first element is the complex response of the MS antenna for a component with vertical polarity, and the second element is the complex response of the MS antenna for a component with horizontal polarity.

Also, matrix \( G_2^{2D} \) is determined as:

\[
G_2^{2D} = \begin{bmatrix} z_i^v \sqrt{\alpha_{m,i}^v} & z_i^h \sqrt{\alpha_{m,i}^h} \\ \exp(j\phi_{m,i}^v) & \sqrt{\alpha_{m,i}^v} \exp(j\phi_{m,i}^h) \\ \exp(j\phi_{m,i}^h) & \sqrt{\alpha_{m,i}^h} \exp(j\phi_{m,i}^v) \end{bmatrix}
\]

(8)

Where, \( G_2^{2D} \) represents random wave components departing from the BS antenna with either \( V \) or \( H \) polarizations and arriving at the MS antenna with \( V \) or \( H \) polarizations. The \( z_i \) terms \( (i=1,\ldots,M) \) in matrix \( G_2^{2D} \) are defined as independent identical distributions (i.i.d.) and are complex exponential variables. \( z_i \) is \( i \)th wave component for each of polarization channels, namely, \( VV, \, VH, \, HV, \, \)and \( VV \). Variables \( \phi_{m,i} \) represent phase offset between wave component departing with \( x \) polarization from the BS and arriving with \( y \) polarization at the MS for \( m \)th \( (m=1,\ldots,M) \) sub-path of \( n \)th \( (n=1,\ldots,N) \) path [4].

Also, the variables \( r_{n1} \) and \( r_{n2} \) are i.i.d. and are described as:

\[
r_{n1} = \frac{P_{vh}}{P_{vv}}, \quad r_{n2} = \frac{P_{hv}}{P_{hh}}
\]

(9)

Where:

- \( P_{vh} \) - power of wave leaving the BS in the vertical direction and arriving at the MS in the horizontal direction.
- \( P_{hv} \) - power of wave leaving the BS in the horizontal direction and arriving at the MS in the vertical direction.
- \( P_{vv} \) - power of wave leaving the BS in the vertical direction and arriving at the MS in the vertical direction.
- \( P_{hh} \) - power of wave leaving the BS in the horizontal direction and arriving at the MS in the horizontal direction.

3.2. Generation of 2-D channel matrix

Channel coefficients are calculated by using the parameters introduced in Section 3.1. The \((u,s)^{th}\) component of the channel coefficients matrix for each of the \( N \) paths is evaluated by [3], [4]:

- \( P_{vh} \) - power of wave leaving the BS in the vertical direction and arriving at the MS in the horizontal direction.
- \( P_{hv} \) - power of wave leaving the BS in the horizontal direction and arriving at the MS in the vertical direction.
- \( P_{vv} \) - power of wave leaving the BS in the vertical direction and arriving at the MS in the vertical direction.
- \( P_{hh} \) - power of wave leaving the BS in the horizontal direction and arriving at the MS in the horizontal direction.
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\[ h_{2D, u, t, n}(t) = \sqrt{\frac{P_d s_t}{M}} \]  
\[ \times \left[ \cos \alpha \sin \cos (\theta_{m, AoA}) \right] \times \left[ \exp \left( j \phi_{h, m} \right) \right] \times \left[ \sqrt{\pi} \exp \left( j \phi_{v, m} \right) \right] \times \sum_{n=1}^{M} \left[ \frac{\cos \alpha}{\sin \cos (\theta_{m, AoA})} \right] \times \exp \left( j 2 \pi d_{i} \sin (\theta_{m, AoA}) \right) \times \exp \left( j 2 \pi \rho \cos (\theta_{m, AoA}) \right) \times \exp \left( j 2 \pi \rho \cos (\theta_{m, AoA} - \theta_{t, r}) \right) \]

Where, parameters used in (10) have been described in previous sections. The superscript 2D indicates wave propagation in two dimensions. Therefore, the \((u,s)^{th}\) component of the channel matrix is channel gain from the \(s^{th}\) element of the MS antenna to the \(u^{th}\) element of the MS antenna and is computed by:

\[ h_{2D, u, t, n}(t) = \sum_{n=1}^{N} h_{2D, u, s, t}(t) \]  

The channel coefficient matrix from the BS antenna to the MS antenna, \(H_{2D}(t)\), can be written as:

\[ H_{2D}(t) = \begin{bmatrix} h_{2D, 1,1}(t) & \cdots & h_{2D, 1,5}(t) \\ \vdots & \ddots & \vdots \\ h_{2D, t,1}(t) & \cdots & h_{2D, t,5}(t) \end{bmatrix}_{t \times s} \]

4. MIMO CHANNEL CAPACITY

In this part, channel capacity is described. If received signal and transmitted signal are shown by \(y\) and \(x\), respectively; then, time-invariant channel is described as:

\[ y = Hx + w \]  

For simplicity, time domain channel variations have been ignored. Dimensions of the vectors \(y\) and \(x\) are \(n_{y} \times 1\) and \(n_{t} \times 1\), respectively. In addition, \(w\) is a zero-mean Gaussian random noise with variance \(N_{0}\) [18].

The capacity is determined by decomposing the channel matrix into a group of parallel and independent scalar Gaussian sub-channels. Using singular value decomposition (SVD), the channel matrix is written as:

\[ H = U \Lambda V^{*} \]

Where, \(U\) and \(V\) are unitary matrices and their dimensions are \(n_{y} \times n_{y}\) and \(n_{t} \times n_{t}\), respectively [1]. Also, \(\Lambda\) is a rectangular matrix with \(n_{y} \times n_{t}\) dimensions that its off-diagonal elements are zero and other elements are real and positive singular values of the matrix \(H\), are arranged as following:

\[ \lambda_{1} \geq \lambda_{2} \geq \cdots \geq \lambda_{n_{t}} \]

By defining new parameters according to:

\[ \tilde{x} = V^{*}x, \quad \tilde{y} = U^{*}y, \quad \tilde{w} = U^{*}w \]

The channel equation is obtained as [1]:

\[ \tilde{y} = \Lambda \tilde{x} + \tilde{w} \]

Where, \(\tilde{w}\) is Gaussian noise and has the same distribution as \(w\). As shown in fig. 2, the representation of the channel is defined as parallel channels in accordance with (17).

Consequently, the total capacity is sum of the capacity of all the parallel and independent sub-channels. Hence, the capacity of the MIMO channel can be computed by using SVD decomposition as following [1]:

\[ C = \sum_{i=1}^{n_{t}} \log \left( 1 + \frac{P_{i}^* \lambda_{i}^{2}}{N_{0}} \right) \text{ bits/s/Hz} \]

Where, \(P_{1}^*, \ldots, P_{n_{t}}^*\) are waterlining powers, selected in a way to achieve maximum capacity according to:

\[ P_{i}^* = \left( \frac{\mu \cdot N_{0}}{\lambda_{i}^{2}} \right)^{\frac{1}{2}}, \quad \sum_{i=1}^{n_{t}} P_{i}^* = P \]

In which, \(\mu\) is used to satisfy the total power.

5. GENETIC ALGORITHM

In this section, the optimization technique that has been used is based on GA. In this method, a key point is to define a fitness function to decode the favorable performance requirements. The fitness function is the MIMO channel capacity.

The basic block of the GA is the chromosome. Each chromosome is composed of genes. The number of genes of each chromosome is equivalent to the number of parallel and independent scalar Gaussian sub-channels. Each gene is equal to the power associated to each of antennas. The summation of genes of each chromosome is fix and equivalent to total power. In fact, we want to distribute total power on antennas in order to get maximum capacity subject to fixed total power.

Steps involved in Genetic algorithm:
Step 1: Initialization – The first step in GA is initialization. We determine the number of sub-channels, number of population size, total power, and etc.

Step 2: evaluation – Based on the objective function the fitness of the antenna powers are computed. In this paper, c.

Step 3: Selection – Based on the fitness function, parents are selected and children are produced.

Step 4: Crossover – We have considered crossover rate of 80% in this algorithm.

Step 5: Mutation – We have considered mutation rate of 20% in this algorithm.

Repeat steps 2 to 5 till the stopping condition is reached.

Stopping condition: It can be taken when average fitness is approximately equal to the maximum fitness or the algorithm can be repeated for a fixed number of generations. Out of the two conditions whichever is achieved first has been taken as the stopping condition. The GA flowchart has been presented in fig. 3.

6. PARTICLE SWARM OPTIMIZATION

In PSO algorithm, each particle moves in a multidimensional space and has memory. All of particles have fitness values which are evaluated by the fitness function to be maximized and they have velocities which lead the movement of the particles. So, the fitness function is MIMO channel capacity. The dimensions of each of particles are equivalent to the number of parallel and independent scalar Gaussian sub-channels. Each particle is equal to the powers associated to the transmitter antennas.

Each particle’s movement is impressed by its local best known position. Also, it is guided toward the best known positions in the search-space, which are updated as better positions are found by other particles. Then, the swarm is moved to the best positions.

PSO is started with a group of particles that are generated randomly. Then, it searches for optimum by updating generations. In every iteration, each particle is updated by two “best” values, named; “pbest and gbest”. The first one is the best fitness that has been achieved so far. The second one is the best fitness that has been obtained so far by any particle in the population. The particles update their velocities and positions after finding these two best values.

Steps involved in PSO:

Step 1: Initialization – the velocity and position of all particles are set randomly subject to satisfy the constraint of total power.

Step 2: Velocity updating – at each iteration, the velocities of all particles are updated.

Step 3: Position updating – at each iteration, the positions of all particles are updated.

Step 4: Memory updating – pbest and gbest are updated.

Repeat steps 2 to 4 till the stopping condition is reached.

Stopping condition: This is such as the stopping condition of GA.

The PSO flowchart has been presented in fig. 4.

7. SIMULATION OF MIMO CHANNEL

By using previously introduced parameters, we have simulated both of the models, un-polarized and cross-polarized antennas, by MATLAB software. Since in each simulation run some of the channel parameters like cluster statistics are varied, we should compare the outputs in the same conditions.

We have computed the MIMO channel capacity, C, using the waterfilling algorithm, GA, and PSO. Then, we have compared them together.

We know that genetic algorithm minimize objective function, C. Since, we want to maximize C, thus, we minimize -C by genetic algorithm.

Using parameters and equations, mentioned in Sections 2 and 3, we have simulated MIMO channel. The parameters values, used for simulation are given in table 1.

<table>
<thead>
<tr>
<th>Table 1. Parameters values used for simulation</th>
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<tr>
<td>parameter</td>
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<tr>
<td>$r, r'$</td>
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<tr>
<td>BS antenna pattern</td>
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<td>AS at BS</td>
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<td>AS at MS</td>
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<tr>
<td>MS antenna gain</td>
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<td>BS antenna gain</td>
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In Fig. 5 and Fig. 6, convergence diagrams of GA and PSO for un-polarized MIMO 4×4 at SNR = +5dB are plotted, respectively. Also, in Fig. 7 and Fig. 8, convergence diagrams of them for cross-polarized MIMO 4×4 at SNR = +5dB are plotted, respectively. These figures show that PSO has better convergence than GA for both of the models, un-polarized and cross-polarized antennas.
**Initialization:**
1. Number of sub-channels \( n_{\text{min}} \) according to (15)
2. Total power \( P \)
3. Number of population size

**Evaluation:**
Computing the MIMO channel capacity (fitness function) subject to fixed total power according to (18) and (19)

**Selection:**
Parents are selected and children are produced based on the channel capacity function according to (18) and (19)

**Crossover:**
Crossover rate: 80%

**Mutation:**
Mutation rate: 20%

**Stopping condition**
Is average fitness approximately equal to the maximum fitness?

Yes

**End**

---

**Initialization:**
1. Number of sub-channels \( n_{\text{min}} \) according to (15)
2. Total power \( P \)
3. Dimensions of each of particles which are equivalent to \( n_{\text{min}} \)
4. Setting randomly the velocity and position of all particles subject to satisfy the constraint of total power according to (19)

**Velocity updating:**
1. Searching for optimum by updating generations.
2. The velocity of each particle is updated by "\( p_{\text{best}} \) and \( g_{\text{best}} \)"

**Position updating:**
1. Searching for optimum by updating generations.
2. The position of each particle is updated by "\( p_{\text{best}} \) and \( g_{\text{best}} \)"

**Memory updating:**
The \( p_{\text{best}} \) and \( g_{\text{best}} \) are updated using computing the MIMO channel capacity (fitness function) according to (18)

**Stopping condition**
Is average fitness approximately equal to the maximum fitness?

Yes

**End**

---

**Fig. 3.** The GA flowchart

**Fig. 4.** The PSO flowchart
Unlike the PSO, we have used the GA toolbox of MATLAB. Then, the programming of both of GA and PSO is different together. As well as, in PSO, any particle is normalized to satisfy the constraint of total power. Using these considerations, PSO has taken lower time than GA for running.

The capacity of the channel has been computed for both of the models, un-polarized and cross-polarized antennas, by using GA, PSO, and waterfilling algorithms and illustrated in fig. 9 to 11, respectively. These figures demonstrate the MIMO channel capacity by using these methods is same and all of them distribute total power on antennas to get maximum capacity subject to fixed total power.

8. CONCLUSION
Numerical examples demonstrate the effectiveness of the PSO and GA. Moreover, the convergence analysis of the PSO and GA algorithms shows that they converge after about 5 and 20 iterations, respectively. Hence, PSO converges faster than the GA.
Due to low computational complexity and their fast convergence, these algorithms are very efficient for
implementation in practical systems such as MIMO channels. These methods indicate good correspondence with the theory's considerations. The simulation results show that these approaches are viable alternatives to existing methods for optimal resource allocation in the spatial MIMO channel.

REFERENCES